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ACCELERATION, COMPRESSION AND STABILITY OF A PLANE LAYER OF MATTER IRRADIATED BY A LASER

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We consider a simple gasdynamic model of acceleration and compression of a plane material layer irradiated by a laser. We establish the conditions under which a limiting isentropic compression takes place, and investigate its stability. We also consider the problem of transfer of the laser radiation energy to the accelerated layer.

A number of experimental and theoretical investigations (see e.g. [1 – 3]) dealt with the problem of transfer of the mechanical recoil impulse to the material target, the impulse resulting from the evaporation and hydrodynamic scattering of the material acted upon by the laser radiation. It is also known that at sufficiently high radiation flux densities, compression waves and in particular shock waves, appear in the nonvaporized material. It is clear that, if the amount of vaporized mass is comparable with the total mass of the target, then the nonvaporized part can be speeded up to velocities approaching that of the flow of the vaporized matter and, under certain conditions, compressed to the densities exceeding appreciably the density of the normal, condensed state. This effect of accelerating low-mass solid targets is of interest in connection with a general problem of accelerating small particles [4] to velocities of 10^5 to 10^7 cm/sec and higher.

1. Let us consider a plane one-dimensional problem of the action of laser radiation with the flux density of q_0, on a plane layer of condensed matter with the initial mass per unit area equal to M. The process of accelerating the layer is determined by the parameters of the material at the vaporization boundary separating the condensed and the gaseous phases, and for this reason the equations of motion must include the gasdynamic laws of conservation of mass flux, impulse and energy at this boundary.

In accordance with [2], in the present case we have

$$\rho_0 D = \rho\,(v + u + D), \quad p_0 = p + \rho_0 D\,(v + u) \tag{1.1}$$

$$q_0{}^* = \rho_0 D\,[\varepsilon + \Omega + {}^1/_2 (v + u)^2] + p\,(v + u), \quad \varepsilon = \frac{p}{(\gamma - 1)\,\rho}$$

where ρ_0 and u $(u > 0)$ are the density and velocity of the layer, p_0 is the pressure in the condensed phase, D is the velocity of the vaporization boundary relative to the layer, v is the velocity of the gas at the vaporization boundary, the direction of irradiation is from right to left, $v > 0$ if the gas is moving to the right and $v < 0$ otherwise; p, ρ and ε are the pressure, density and specific internal energy of the gas at the vaporization boundary, Ω is the specific bond energy of the material, γ is the adiabatic index and $q_0{}^*$ is the radiation flux density at the vaporization boundary.

The equations describing the motion of the nonvaporized part of the layer of mass M have the form

$$M\frac{du}{dt} = p + \rho_0 D\,(v + u), \qquad \frac{dM}{dt} = -\,\rho_0 D \tag{1.2}$$

The system (1.1),(1.2) is a general one, and independent of the character of the gasdynamic motion of the vaporized material, i.e. it is independent of the mode of vaporization.

In the present case the third equation of (1.1) which expresses the law of conservation of energy at the discontinuity (vaporization boundary) can be reduced, with the help of the first two equations of (1.1), to the form

$$q_0{}^* = q_h + \frac{d}{dt}\left(\frac{Mu^2}{2}\right) \tag{1.3}$$

where q_h is the hydrodynamic energy flux of the vaporized material. Thus, unlike the equation in [2], the present equation contains a term describing the transfer of radiation energy to the kinetic energy of the layer.

The complete problem of acceleration of the nonvaporized part of the layer must include the hydrodynamics of the dispersing gaseous phase, i.e. the system consisting of the first two equations (1.1) – (1.3) must be supplemented, in general, with a system of hydrodynamic equations which take into account the absorption of laser radiation and electronic heat conductivity. Such a problem can only be solved accurately by numerical methods. We can use the equations given above to study a simple physical model, which will enable us to obtain the basic relations describing a process by which the laser radiation energy is transferred to the kinetic energy of the layer.

Indeed, let us assume, that in a coordinate system attached to the vaporization boundary the vaporization takes place by a steady state mechanism, i.e. the quantities p, ρ, D, q^* and $v + u + D$ are time independent. In this case the following Jouguet condition holds [5]:

$$v + u + D = c = (\gamma p\,/\,\rho)^{1/2}$$

where c is speed of sound at this boundary. Under this condition the system (1.2) gives (with $\rho \ll \rho_0$ and $p\,/\,\rho \gg \Omega$)

$$M = M_0 - \rho_0 D t, \qquad u = \frac{p_0}{\rho_0 D} \ln \frac{M_0}{M} \tag{1.4}$$

$$T = \frac{Mu^2}{2} = \frac{1}{2}\left(\frac{\gamma + 1}{\gamma}\right)^2 c^2 M \ln^2 \frac{M_0}{M}$$

$$E_0{}^* = q^* t = (M_0 - M)\,c^2\left[\frac{1}{\gamma\,(\gamma - 1)} + \frac{1}{2} + \frac{1}{\gamma}\right]$$

where T is the kinetic energy of the condensed part of the layer and $E_0{}^*$ is the energy transported to the vaporization boundary. From the third and fourth relation of (1.4) we obtain

$$\eta_0(t) = \frac{T}{E_0{}^*} = \frac{1}{2}\left(\frac{\gamma+1}{\gamma}\right)^2\left[\frac{1}{\gamma(\gamma-1)} + \frac{1}{2} + \frac{1}{\gamma}\right]^{-1}\frac{M}{M_0 - M}\ln^2\frac{M_0}{M}$$

The function $\eta_0(t) = \eta(M_0 / M)$ attains its maximum value when $M / M_0 = 0.2$ and is then equal to $\eta_{0\max} = 0.64(1 - 1/\gamma^2) \approx 41\%$ $(\gamma = {}^5/_3)$.

Thus, $\eta_0 = 41\%$ is the limiting value of the coefficient of conversion of the laser radiation energy into the kinetic energy of the layer. This value can be attained when all radiant energy is delivered to the hydrodynamic discontinuity $(q_0{}^* = q_0)$. This is not, however, realized in practice. Some of the energy is consumed in heating the outer, vaporized part of the layer. In the plane case this screening effect causes a certain amount of the vaporized matter to move in the same direction, as the condensed part of the layer.

2. The authors of [6] give the results of a numerical analysis of the limiting isentropic compression of matter under the action of pulsed laser radiation. Realization of such a mode involves a laser pulse of specified profile with respect to time.

In the present paper we consider an analytic model of isentropic compression of a plane layer of matter. We obtain the spatio-temporal distributions of the hydrodynamic parameters within the layer, and this enables us to formulate the requirements concerning the temporal form of the pressure impulse $p_0(t)$ and, consequently, the form of the laser radiation pulse necessary for the realization of the isentropic compression mode.

Let the initial state of the layer of thickness x_0 be specified by the parameters ρ_0 and c_0, where c_0 is the speed of sound. The acceleration and compression of the layer under the action of a pressure impulse will be described in the acoustic approximation, without taking into account the dissipative processes. In this case the system of hydrodynamic equations has the form [5]

$$\frac{\partial}{\partial t}\left[v + \frac{2}{\gamma-1}c\right] - (v+c)\frac{\partial}{\partial x}\left[v + \frac{2}{\gamma-1}c\right] = 0,\quad v = \frac{2}{\gamma-1}(c - c_0) \qquad (2.1)$$

Here the coordinate $x > 0$ is measured from the inner boundary of the layer, while $v(x)$ and $c(x)$ are the velocity of the medium and speed of sound at the point x.

The above equations describe the mode of compression without shock waves. For this reason the characteristic time dimension of the problem is $t_0 = x_0/c_0$, and it corresponds to the time of arrival of the first perturbation at the inner boundary. Substituting the second equation of (2.1) into the first, we obtain

$$\frac{\partial c}{\partial t} - \left[\frac{\gamma+1}{\gamma-1}c - \frac{2}{\gamma-1}c_0\right]\frac{\partial c}{\partial x} = 0 \qquad (2.2)$$

We shall seek the solution of (2.2) in the form

$$c = c_0 C(\lambda), \qquad \lambda = \frac{x}{x_0(1 - t/t_0)} \qquad (2.3)$$

The value $\lambda = 1$ corresponds to the trajectory of the first perturbation. If a continuous solution of the form (2.3) exists, then it will correspond to the isentropic compression of the layer, since the discontinuities, i.e. the shock waves, can appear behind the boundaries of the layer of thickness x_0 only when $t > t_0$.

Substituting (2.3) into (2.2), we obtain an equation, the nontrivial solution of which is

$$C = \frac{\gamma - 1}{\gamma + 1}\lambda + \frac{2}{\gamma + 1}$$

The expression for the remaining hydrodynamic parameters are obtained with the help of the well known hydrodynamic relations

$$\frac{c}{c_0} = \frac{\gamma - 1}{\gamma + 1}\left[\lambda + \frac{2}{\gamma - 1}\right], \quad \frac{v}{c_0} = \frac{2}{\gamma + 1}\left[\frac{\gamma - 1}{\gamma + 1}\lambda - 1\right] \tag{2.4}$$

$$\frac{\rho}{\rho_0} = \left[\frac{\gamma - 1}{\gamma + 1}\left(\lambda + \frac{2}{\gamma - 1}\right)\right]^{2/(\gamma-1)}, \quad \frac{p}{\rho_0 c_0^2} = \frac{1}{\gamma}\left[\frac{\gamma - 1}{\gamma + 1}\left(\lambda + \frac{2}{\gamma - 1}\right)\right]^{2\gamma/(\gamma-1)}$$

where at the given instant of time t

$$x_0 (1 - t / t_0) \leqslant x < x_1 (t)$$

Here $x_1 (t)$ is the trajectory of the outer boundary of the layer, and can be found from the equation $dx_1 / dt = - v (x_1)$.

Since $x_1 = \lambda_1 x_0 (1 - t / t_0)$, we obtain the following relation for λ_1:

$$1 - \frac{t}{t_0} = \exp\left\{-\int_1^{\lambda_1} \frac{d\lambda}{\lambda - u(\lambda)}\right\}, \quad u(\lambda) = \frac{v}{c_0}$$

which, together with the second relation of (2.4) gives

$$x_1 (t) = x_0 \frac{\gamma + 1}{\gamma - 1}\left[\left(1 - \frac{t}{t_0}\right)^{2/(\gamma+1)} - \frac{2}{\gamma + 1}\left(1 - \frac{t}{t_0}\right)\right] \tag{2.5}$$

It can be shown that the solution obtained which contains the trajectory $x_1 (t)$ of "a piston", corresponds to the situation in which the perturbations generated by the piston converge at the inner boundary of the layer simultaneously, at the time $t = t_0$. Indeed, the law of motion $x_1 (t)$ can be obtained in the manner analogous to that in [7], from the equation

$$\frac{x_1 (t)}{v_1 (t) + c (x_1)} = dt + \frac{x_1 (t) - v_1 (t)\, dt}{v_1 + c (x_1) + dc + dv_1}$$

or

$$\frac{\gamma + 1}{2} x_1 \frac{dv_1}{dt} = v_1 \left[\gamma c_0 + \frac{\gamma^2 - 1}{4} v_1\right] + c_0^2$$

Integration of the last equation gives (2.5).

Next we shall show that the solution of the present problem in the two-dimensional case, with the law of motion of the piston given, can be obtained using the method of characteristics, with the help of the general integral of the initial equation (2.2)

$$x + \left[\frac{\gamma + 1}{\gamma - 1} c - \frac{2}{\gamma - 1} c_0\right] t = \varphi (c)$$

where $\varphi (c)$ is a function which can be determined using the given law of motion of the piston. However, the method of solution developed in the present paper can be generalized to the cases of the spherical and cylindrical geometries, and for these cases the method of characteristics cannot be used. Using the relations (2.4), we obtain the following expressions for the hydrodynamic parameters at $x = x_1 (t)$

$$\frac{c}{c_0} = \left(1 - \frac{t}{t_0}\right)^{-(\gamma-1)/(\gamma+1)}, \quad \frac{\rho}{\rho_0} = \left(1 - \frac{t}{t_0}\right)^{-2/(\gamma+1)}, \quad \frac{p_0}{\rho_0 c_0^2} = \frac{1}{\gamma}\left(1 - \frac{t}{t_0}\right)^{-2\gamma/(\gamma+1)} \tag{2.6}$$

The last expression in (2.6) defines the temporal form of the pressure impulse at the outer boundary of the layer, at which the compression mode is nearly isentropic. Using (2.6) we can find the temporal form of the laser impulse which is necessary for the realization of such a mode. Indeed, the magnitude of the hydrodynamic pressure $p_0(t)$ transmitted to the region of dense matter under the action of radiation is connected with the radiation flux density $q_0(t)$ and the speed of sound $c_0(t)$, by the expression [2]

$$p_0(t) \sim q_0(t) / c_0(t) \tag{2.7}$$

From (2.6) and (2.7) we find

$$q_0(t) \sim (1 - t / t_0)^{-(3\gamma-1)/(\gamma+1)} \tag{2.8}$$

which differs somewhat from the corresponding expression given in [5] where $q_0(t) \sim (1 - t / t_0)^{-3\gamma/(\gamma+1)}$.

The last expression can be obtained under the assumption that the dynamics of the dispersion of the outer part of the layer (corona) and, consequently, the inward transmission of the pressure impulse, depends on the characteristic density ρ near which the electronic heat flux is transformed into the hydrodynamic flux of the dispersing corona

$$p \sim \rho c^2, \quad q \sim \rho c^3, \quad p \sim q^{2/3}, \quad q \sim (1 - t / t_0)^{-3\gamma/(\gamma+1)}$$

From (2.4) it follows that when $t \to t_0$, then the pressure, the density, the work done above the layer and the kinetic energy, all tend to infinity. Obviously, the physical quantity characteristic of the process is the ratio of the kinetic energy of the layer to the total work done above this layer. This quantity naturally remains finite when $t \to t_0$, and is equal to $2\gamma/(3\gamma - 1) = 83\%$ $(\gamma = 5/3)$.

3. Let us investigate the stability of the solutions (2.4) obtained, when the boundary or the initial conditions undergo small perturbations. In the present case it is sufficient to investigate the behavior of the perturbations affecting one of the hydrodynamic quantities. The remaining ones can then be calculated by means of the thermodynamic relations. Let us consider one-dimensional perturbations in the speed of sound. Let

$$c' = c + \Delta(x_1 t) \tag{3.1}$$

where c is the unperturbed solution given by (2.4). Substituting (3.1) into (2.2) and linearizing in the usual manner, we obtain

$$\frac{\partial \Delta}{\partial t} - \left[\frac{\gamma+1}{\gamma-1} c - \frac{2}{\gamma-1} c_0\right] \frac{\partial \Delta}{\partial x} - \frac{\gamma+1}{\gamma-1} \Delta \frac{\partial c}{\partial x} = 0 \tag{3.2}$$

which can be solved when either the initial conditions $\Delta(x,0) = \Delta_0(x)$ or the boundary conditions $\Delta_0(0, t) = \Delta_0(t)$ are given. Substituting c from (2.4) into (3.2) and introducing the variable $\theta = 1 - t / t_0$, we reduce it to the form

$$\frac{\partial \ln \Delta}{\partial \ln \theta} + \frac{\partial \ln \Delta}{\partial \ln x} + 1 = 0 \tag{3.3}$$

The general solution of (3.3) is

$$\Delta(x, t) = \frac{1}{x} \Phi\left(\frac{1 - t / t_0}{x}\right) \tag{3.4}$$

where Φ is an arbitrary function the form of which is determined by the boundary or the initial conditions.

The general form of (3.4) already suggests that the arbitrary perturbations increase with time on the first characterisitcs faster than the basic solution. Indeed, substituting into (3.4) $x = x_0(1 - t/t_0)$, we obtain

$$\Delta(x, t) = \frac{\Phi(1/x_0)}{x_0(1 - t/t_0)}$$

while the maximum rate of increase of the basic solution is, according to (2.4), $c \sim (1 - t/t_0)^{(\gamma-1)/(\gamma+1)}$. The effect of the perturbation on the character of the motion depends on the sign of the perturbation. Let us recall the interpretation and the character of the Riemann solution of which (2.4) is a particular case. The fastest-moving particles are those at the piston, their velocity falls in the direction of motion and reaches the value equal to the initial speed of sound in the medium, at the first characteristic. The pressures and the density are distributed analogously (see (2.4)). On approaching the point of intersection of the characteristics the "piston" catches up with the first characteristic and the curvature of the front increases. Finally, after crossing the characteristics the solution becomes multivalued and invalid, and is replaced by a discontinuous solution. In the present case all quantites become divergent at this particular moment.

Let us now investigate the various forms of the initial and boundary conditions.

3.1. Initial spatial distortion of the speed of sound. This situation can occur in practice when the properties of the medium vary slightly along the coordinate (e.g. temperature variation along x), $\Delta_0(0, x) = \alpha c_0 \cos k_0 x$, $\alpha \ll 1$. Then the arbitrary function can be found from the condition

$$\frac{1}{x}\Phi(x) = \alpha c_0 \cos k_0 x$$

and in this case the solution is obtained in the form

$$\Delta(x, t) = \alpha c_0 \cos\left(\frac{k_0 x}{1 - t/t_0}\right)\left(1 - \frac{t}{t_0}\right)^{-1}$$

When $t = t_0$, this expression diverges and, since the perturbations at any point enveloped by the motion grow faster than the unperturbed solution, their ratio Δ / c also becomes divergent when $t \to t_0$. The solutions (2.4) – (2.6) are unstable and the random perturbations in this model lead to formation of shock waves before maximum compression is reached.

3.2. A small perturbation is given at the piston trajectory, proportional to the unperturbed solution, $\Delta_0(x, t/t_0) = \alpha c_1(t/t_0)$, $\alpha \ll 1$. In this case the solution is

$$\Delta(x, t) = \frac{\alpha c_0}{1 - t/t_0}\left[\frac{2}{\gamma+1}\left(1 + \frac{(\gamma-1)x}{x_0(1 - t/t_0)}\right)\right]^{-2/(\gamma-1)}$$

from which it follows that on the first characteristic $\Delta/c \sim (1 - t/t_0)^{-1}$, i.e. it diverges as $t \to t_0$.

3.3. Initial perturbation at the piston finite at the initial moment and rapidly decreasing with time, $\Delta_0(x, t/t_0) = \alpha c_1(t/t_0)(1 - t/t_0)^n$, $n > 1$, $\alpha \ll 1$. The complete solution is

$$\Delta(x, t) = \frac{\alpha c_0}{1 - t/t_0}\left[\frac{2}{\gamma+1}\left(1 + \frac{(\gamma-1)x}{2x_0(1 - t/t_0)}\right)\right]^{-(\gamma+1)(n-1)/(\gamma-1)}$$

In the last two cases the solution remains continuous despite the fact that the ratio Δ/c

diverges on the first characteristic. This results from the fact that the velocity of the medium increases near the first characteristic, with the particles "escaping" from the piston. Apparently this type of stability leads to the transformation of the solution (2.4) into another, Riemann-type solution, different from the unperturbed solution, in which the characteristics either intersect at a later instant, or do not intersect at all at a general point.

When an attempt is made to realize, in practice, a mode resembling (2.4), small chaotic perturbations of different sign and amplitude appear invariably in the initial and boundary conditions. These perturbations are caused by the deviation of the real conditions from ideality. The development of perturbations of the same sign as the velocity of the piston imparts an acceleration to some particles, and those of the opposing sign retard other particles. Thus the growth of perturbations speeds up the appearance of ambiguity in the solutions, i.e. the formation of discontinuities. The solutions (2.4) are valid on the initial interval of the motion. The presence of small chaotic perturbations causes subsequent appearance of the shock waves. Apparently this argument also holds for other Riemann-type solutions leading to infinite compression; in particular the solutions of this type obtained by numerical methods [6] can also become unstable in the sense indicated above.

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